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THE ADEQUACY OF A SHEAR BUILDING FOR MODELLING OPEN-PLANE FRAMES UNDER SEISMIC EXCITATION

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1. INTRODUCTION

While the dynamic analysis of framed structures can be performed on the basis that they have continuously distributed properties, the analyses are complex and practicable only in the case of very simple structures. More conveniently, a finite element based approach can be adopted in which framed structures are discretized into segments and the displacements of the interconnecting nodes constitute the generalized co-ordinates (or dynamic degreess of freedom) of the structure. The number of one-dimensional elements selected depends on the physical arrangement of the structure. Not all of the degrees of freedom need be considered in determining the response to an arbitrary dynamic load. Kinematic constraints are adopted to reduce the degrees of freedom in order to save computational effort without significant loss of accuracy.

As an extreme, for framed structures it is usually assumed that the floor slabs have considerable in-plane rigidity and that the columns are inextensible. The mass is assumed to be concentrated at the floors and to possess only translatory degrees of freedom. An n-story plane frame thus has only n degrees of freedom along its plane. The structure is then known as a *shear building* and is commonly used to model structures subjected to horizontal ground motion. The computational effort is reduced considerably when this form of discretization is adopted in a dynamic analysis. Also, a great reduction in the computer memory requirement is achieved, since an n-story tall plane framed structure has only n degrees of freedom in the plane of deformation considered, irrespective of the number of bays of which it is comprised. Since mode superposition is feasible in an elastic analysis, the shear building representation also permits use of an approximate analysis of the response to seismic excitation based on ground motion response spectra.

The adoption of the shear building discretization for the analysis of tall framed structures subjected to horizontal seismic excitation is not only very common in practice but is also found in the literature, and the topics include the effects of soil–structure interaction on the dynamic behavior of such structures [1, 2], the effects of torsional coupling on earthquake forces in buildings [3, 4] the optimum design of steel frames under earthquake loading [5] and the study of the along wind motion of multi-storey buildings [6].

It can be anticipated that shear building discretization will result in some difference in the fundamental period as compared to a more realistic finite element based model. Furthermore, the fundamental mode of vibration for the shear building is a very simplistic approximation of the elaborate vibration shape yielded by a finite element representation. These two features could result in the response yielded by the shear building differing from that for the more exact finite element model.

In this note, we examine the appropriateness of representing open-plane frame structures subjected to horizontal seismic excitation by the shear building model when it is feasible

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(at the cost of a little extra storage requirement and computational effort) to treat such frames as assemblages of one-dimensional finite elements.

2. THE FRAMES STUDIED

In Figure 1(a) are shown the plans of typical open-framed reinforced concrete long structures of flat slab construction, wherein transverse frames can be isolated, as shown shaded, and analyzed for the response to a horizontal ground motion transverse to the longitudinal axis. For each structure, the slabs with columns constitute flexible frames which are rigidly fixed at the base.

For the purpose of this study, it was assumed that the transverse frames were spaced at 4·0 m centers and six typical frames were considered, viz., four- and eight-storey frames each having one, two and three bays. Some typical four-storey frames are shown in Figure 1(b). For all of the frames, a uniform inter-storey height of 3 m and bay span of 6 m were adopted. Furthermore, each slab was taken to be 0·3 m thick and 4 m wide, and the column sections were uniform over the height of the structure. For four-storey frames, column sections *bd* of 0·32 m × 0·32 m, 0·3 m × 0·4 m and 0·2 m × 0·5 m were used, where *b* is the transverse dimension and *d* is the in-plane dimension, in order to have *EI* (flexural rigidity) values ranging between 19·2 MN m^2 and 45·8 MN m^2. For eight-storey frames, *bd* values of 0·3 m × 0·4 m, 0·2 m × 0·5 m and 0·3 m × 0·6 m were adopted (*EI* varying from 35·2 MN m^2 to 118·8 MN m^2). Linear elastic analysis was performed with the following material properties: modulus of elasticity of concrete, $E_c = 22$ GPa and mass density, $\rho = 2400$ kg/m^3.

To examine the adequacy of the shear building for representing open-plane frames, a finite element type of representation of each frame was adopted as the standard. Nodes were located at column–slab junctions and at the mid-spans of slabs. Each node possesses three degrees of freedom. The elements have uniformly distributed inertia and elastic properties. Cubic Hermitian polynomials were used to evaluate the mass and stiffness

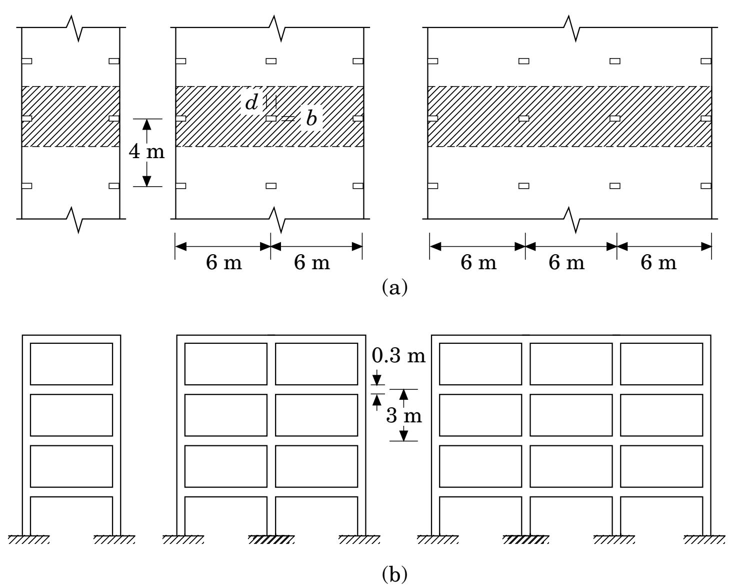

Figure 1. The plan and elevation of typical open-plane frames considered: (a) plan; (b) elevation of typical four-storey frames.

coefficients. The system has a consistent mass matrix. This discretization is hereafter referred to as consistent mass (CM) model.

In the shear building (SB) model, the mass of the structure was lumped at the floor levels. The stiffnesses of the columns at each floor level were added together and the stiffness matrix of the SB model was derived assuming that only horizontal displacements of the floors occur.

3. DETAILS OF DYNAMIC LOADS SELECTED AND THE ANALYSIS

The first 30 s of the horizontal acceleration components of five earthquakes were used as seismic loading to evaluate the performance of the SB model of each structure against that of the more refined CM model. Details of the accelerograms used are presented in Table 1.

The analysis consisted of determining the eigenvalues and eigenvectors for the undamped system using the Jacobi method. For each typical structure, the response of the CM model and the SB model to the seismic excitations selected was obtained in the time domain (using modal analysis) by evaluating the Duhamel integral, and the results are reported in the form of peak lowest storey column shears. The damping ratio was constant for all modes of vibration. Three magnitudes of damping ratio were adopted, viz. 5%, 10% and 15%, to cover the range of damping encountered in practice reference [7].

For the SB model, all of the modes of vibration were considered in evaluating the response. The corresponding CM model possesses a far larger number of dynamic degrees of freedom. However, it is well recognized that just a few of the lowest modes of vibration are adequate to define the response of a structure. In particular, computations for a four-storey one-bay frame subjected to seismic excitation Q5 with a damping ratio of 5% indicated that the fundamental mode alone yields 95% of the response for the SB model and about 96% for the CM model. However, to determine the response of any frame to the excitation using the CM model, the number of lowest order modes used was restricted to the number of degrees of freedom, n, of the corresponding SB model.

TABLE 1

Details of the earthquakes selected

Description of earthquakes		Maximum acceleration		Spectral pseudo-velocity for $\xi = 5\%$		Response spectrum intensity for $\xi = 5\%$, SI (m)
Description	Symbol	Value (m/s^2)	Time (s)	Max. S_v (m/s)	Period (s)	
Uttarakashi, 1991 (N15°W)	Q1	−2·372	6·22	0·464	0·249	0·432
Uttarakashi (Abhat), 1991 (N85°E)	Q2	2·484	4·26	0·541	0·887	0·697
Eurake, 1954 (N46°W)	Q3	1·973	7·10	0·697	1·413	1·038
El Centro, 1940 (S90°W)	Q4	2·101	11·44	0·724	2·067	1·119
El Centro, 1940 (S00°E)*	Q5	3·417	2·12	0·809	1·331	1·331

*Also known as NS component of El Centro. The notation used here is also seen in reference [8].

4. RESULTS AND DISCUSSION

4.1. *Periods of vibration and mode shapes*

In Table 2 are indicated the four largest periods of vibration for four-storey and eight-storey frames, having one, two and three bays for column dimensions of 0·2 m × 0·5 m ($EI = 45{\cdot}8$ MN m^2). Irrespective of the number of storeys or bays, the SB model yielded smaller periods of vibration for all of its modes compared to the CM model. This may be due to the fact that the stiffness matrix for the shear building is evaluated taking account only of the translational degrees of freedom of the structure. In both discretizations, the fundamental period increases as the number of bays increases.

It is also seen in Table 2 that the ratio of fundamental period (T_{sb}) of the SB model to that of the CM model (T_{cm}) varies non-uniformly with the number of bays, but is always less than unity. The period ratio was also determined for a practical range of column EI values (19·2–45·8 MN m^2 for four-storey frames and 35·2–118·8 MN·m^2 for eight-storey frames). As seen in Figure 2, for any type of frame the period ratio increases with the period of the CM mdel but, due to practical limitations in the choice of EI values, it does not exceed 0·9 for the four-storey frame and 0·811 for the eight storey frame. Thus the SB model does not yield a comparable fundamental period over a practical range of column stiffness for the frames considered.

In Figure 3 are shown the fundamental modes of vibration of the SB and CM models of a four-storey one-bay frame. Compared to the SB model, the fundamental mode of the CM model shows small rotations, apart from storey translations. As reported earlier, the fundamental mode of vibration accounts for over 95% of the maximum response to seismic excitation. It is likely that differences in the fundamental period and in the mode shape may lead to the results of seismic analysis based on the SB model being greatly at variance with those obtained when a more realistic finte element type discretization is used.

4.2. *Response to seismic excitation*

It is common to report the results of seismic analysis in terms of relative horizontal displacements at the locations of interest. However, the member end actions mobilized in the columns are functions of both relative nodal displacements and nodal rotations which

TABLE 2

Periods (s) of four- and eight-storey frames (column $EI = 45{\cdot}8$ MN m^2)

	One-bay frame		Two-bay frame		Three-bay frame	
Mode no.	SB	CM	SB	CM	SB	CM
Four-storey frame						
1	0·385	0·511	0·395	0·558	0·466	0·576
2	0·134	0·166	0·139	0·184	0·162	0·191
3	0·088	0·097	0·093	0·111	0·106	0·116
4	0·072	0·085	0·079	0·091	0·087	0·093
T_{sb}/T_{cm}	0·753		0·708		0·809	
Eight-storey frame						
1	0·727	1·068	0·832	1·119	0·880	1·140
2	0·245	0·345	0·281	0·371	0·297	0·377
3	0·151	0·194	0·172	0·217	0·182	0·221
4	0·112	0·136	0·128	0·157	0·135	0·161
T_{sb}/T_{cm}	0·681		0·744		0·772	

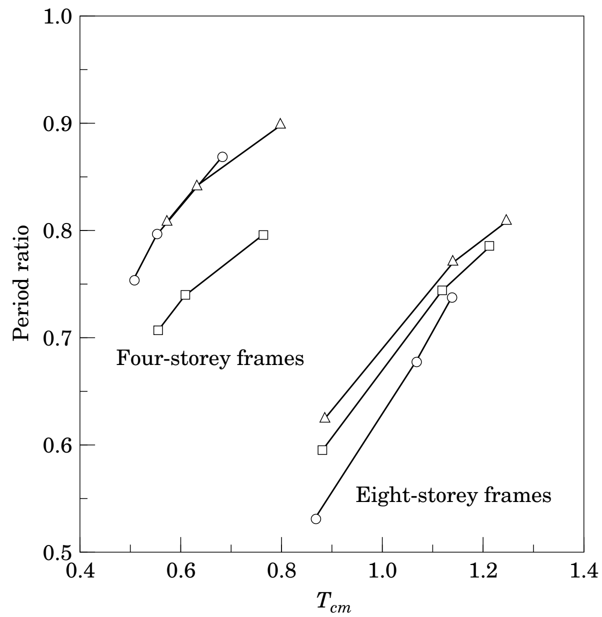

Figure 2. The relationship between the period ratio and the period of the building. —○—, one-bay frame; —□—, two-bay frame; —△—, three-bay frame.

can amplify or attenuate the effect of relative displacements. The effect of seismic excitation is therefore reported here in terms of column shear mobilized.

The maximum lowest storey shear and the duration of excitation at which it is mobilized is shown in Table 3(a) for four-storey structures of one, two and three bays for the five accelerograms adopted, when the structures have a column flexural rigidity of 45·8 MNm^2. These results pertain to a constant modal damping ratio of 5%. The data in Table 3(b) refers to eight-storey structures having the same column *EI* value and damping ratio. It

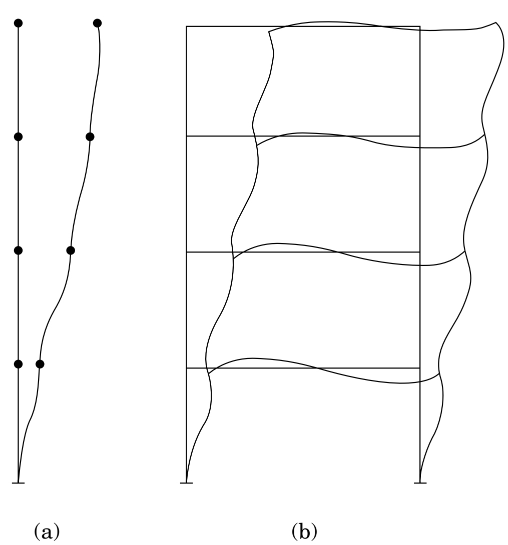

Figure 3. Fundamental mode shapes for a four-storey one-bay structure: (a) SB model; (b) CM model.

TABLE 3

Maximum absolute shear (*Column* $EI = 45{\cdot}8$ MNm^2)

(a) Four-store structures

Excitation	Model	One-bay Force (kN)	One-bay Time (s)	Two-bay Force (kN)	Two-bay Time (s)	Three-bay Force (kN)	Three-bay Time (s)
Q1	CM	119·05	6·70	112·82	7·00	117·36	7·00
	SB	188·05	5·80	212·70	5·80	179·66	5·90
Q2	CM	170·14	4·70	195·93	5·80	233·70	5·90
	SB	122·58	5·70	149·75	5·70	229·52	4·40
Q3	CM	114·88	7·10	150·46	8·00	172·12	7·20
	SB	99·22	6·70	111·19	6·70	152·52	7·10
Q4	CM	205·23	2·10	219·09	2·10	245·24	11·90
	SB	168·49	11·50	214·13	11·50	252·85	2·10
Q5	CM	268·66	2·40	247·28	2·20	378·14	2·20
	SB	211·22	5·00	230·72	5·00	419·53	5·10

(b) Eight-storey structures

Excitation	Model	One-bay Force (kN)	One-bay Time (s)	Two-bay Force (kN)	Two-bay Time (s)	Three-bay Force (kN)	Three-bay Time (s)
Q1	CM	73·48	4·40	101·20	5·60	114·64	5·60
	SB	216·49	6·00	199·55	6·10	179·75	6·10
Q2	CM	114·37	8·20	122·80	7·70	128·66	7·70
	SB	191·30	4·90	363·02	6·80	403·46	6·40
Q3	CM	217·41	8·00	264·24	8·00	285·59	8·00
	SB	260·79	7·20	360·51	7·30	375·06	7·30
Q4	CM	166·00	12·80	178·20	12·20	205·21	12·20
	SB	292·04	12·00	304·20	12·10	295·30	12·10
Q5	CM	243·23	4·40	261·75	3·00	278·69	3·00
	SB	364·62	2·70	492·30	5·80	478·71	5·90

can clearly be seen that the response yielded by the SB model varies significantly from that yielded by the CM model. The excitation duration at which it occurs also differs. Although the fundamental period of the SB model is always less than that of the corresponding CM model, the maximum lowest storey column shear yielded by the SB model can be more or less than that yielded by the CM model. This indicates that the frequency content of the selected excitation is important at low damping ratios (5%) where small changes in period can yield widely different pseudo-spectral velocity (S_v) values, as can be seen in Figure 4. As a result, for any particular frame the response ratio (which is defined as the ratio of maximum absolute first storey shear of the SB model to that of the CM model) is less than or greater than 1·0 depending on the frequency content (pseudo-spectral velocity spectrum) of the excitation. As seen in Table 4(a), the response ratio is rarely close to unity.

For higher levels of damping, the response spectrum of an excitation is smoothed out, as can be seen in Figure 4, in which the response spectra for Q2 for damping ratios of

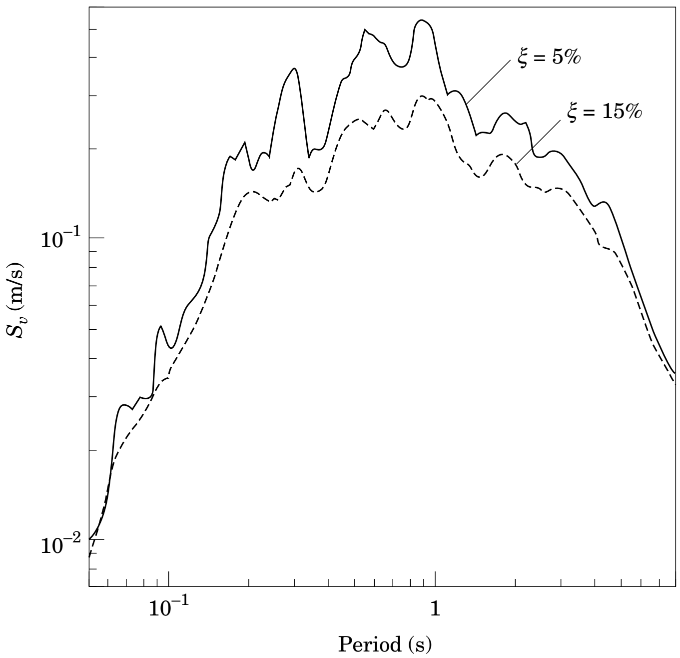

Figure 4. Response spectra of earthquake Q2 ($\xi = 5\%$ and $\xi = 15\%$).

5% and 15% are presented. As a result, for higher levels of damping small differences in fundamental period should not result in much variation in S_v. However, as seen from Tables 4(b) and 4(c), which are for damping ratios of 10% and 15% respectively, the response ratio is not influenced very significantly by the damping level. Therefore, while the S_v values can be quite close, the differences in fundamental mode shapes of the SB model and the CM model result in the response ratio not being unity. The SB model ignores the rotary inertia forces which are generated in the more realistic CM model.

On the other hand, except for the storage requirements of the larger mass and stiffness matrices required for the CM model, and the additional computational time needed when working with large matrices to extract the eigenvalues and eigenvectors, there is no significant saving in computational time when the shear building is used instead of the CM model (with the number of modes used in the latter being restricted to the number of floors). It is therefore questionable whether using the SB model and obtaining results which rarely concur with those yielded by the CM model can be justified.

5. CONCLUSIONS

A shear building yields a stiffer system, as the fundamental frequency is always higher than that of the consistent mass (CM) model, irrespective of the number of storeys and the number of bays. This is chiefly due to only translatory degrees of freedom being considered in evaluating the stiffness of the system. While the difference in fundamental frequency yielded by both models can be reduced by having more flexible columns, due to practical considerations, the shear building generally has a fundamental frequency over 10% higher than that of the CM model.

The difference in the fundamental period and in the fundamental mode shape result in responses evaluated using the shear building (SB) model varying considerably from those yielded by a more refined finite element based model for low damping ratios.

The variation in responses obtained using a shear building and consistent mass models for analysis persists even for large damping ratios.

The results show that the shear building model is a very unrealistic representation of framed structures subjected to horizontal seismic excitation.

TABLE 4

Response ratios of frames due to earthquake excitations

(a) $\xi = 5\%$

Period of shear building, T_{sb}	Period ratio, T_{sb}/T_{cm}	Frame type†	Response ratio for excitations				
			Q1	Q2	Q3	Q4	Q5
0·385	0·753	4-1C	1·58	0·72	0·86	0·82	0·79
0·395	0·708	4-2C	1·88	0·97	0·78	0·98	0·82
0·442	0·796	4-1B	1·68	0·87	0·71	0·88	0·76
0·453	0·740	4-2B	1·43	1·36	0·96	1·37	1·15
0·464	0·533	8-1D	1·47	1·05	0·85	1·83	1·79
0·466	0·809	4-3C	1·53	0·98	0·90	1·03	1·11
0·528	0·597	8-2D	1·72	1·50	1·14	2·32	1·93
0·534	0·841	4-3B	1·30	1·50	1·15	1·39	1·37
0·557	0·627	8-3D	1·55	1·48	1·18	2·12	2·13
0·595	0·869	4-1A	0·83	1·55	1·05	1·30	1·53
0·610	0·793	4-2A	1·08	1·39	0·99	1·17	1·24
0·721	0·900	4-3A	1·17	0·92	1·03	1·17	1·17
0·727	0·681	8-1C	2·95	1·67	1·20	1·76	1·50
0·832	0·744	8-2C	2·05	2·95	1·36	1·71	1·89
0·835	0·736	8-1B	1·79	2·83	1·27	1·51	1·82
0·880	0·772	8-3C	1·57	3·14	1·31	1·44	1·72
0·954	0·787	8-2B	1·00	2·71	1·26	0·87	1·79
1·009	0·811	8-3B	0·89	2·04	1·22	1·01	1·95

(b) $\xi = 10\%$

Period of shear building, T_{sb}	Period ratio, T_{sb}/T_{cm}	Frame type†	Response ratio for excitations				
			Q1	Q2	Q3	Q4	Q5
0·385	0·753	4-1C	1·79	0·72	0·84	0·88	0·71
0·395	0·708	4-2C	1·93	1·01	0·82	0·99	0·84
0·442	0·796	4-1B	1·73	1·00	0·76	0·89	0·78
0·453	0·740	4-2B	1·36	1·32	0·92	1·42	1·05
0·464	0·533	8-1D	1·93	1·25	0·87	1·74	1·60
0·466	0·809	4-3C	1·54	1·27	0·90	1·15	0·94
0·528	0·597	8-2D	1·88	1·48	1·03	2·19	2·11
0·534	0·841	4-3B	1·17	1·32	0·99	1·43	1·31
0·557	0·627	8-3D	1·81	1·43	1·19	2·07	2·37
0·595	0·869	4-1A	0·82	1·20	1·01	1·14	1·47
0·610	0·793	4-2A	1·11	1·25	1·07	1·27	1·28
0·721	0·900	4-3A	1·27	0·91	1·08	1·22	1·18
0·727	0·681	8-1C	2·84	1·75	1·38	1·65	1·55
0·832	0·744	8-2C	2·14	2·71	1·46	1·47	1·59
0·835	0·736	8-1B	1·85	2·58	1·35	1·35	1·51
0·880	0·772	8-3C	1·68	2·95	1·46	1·32	1·57
0·954	0·787	8-2D	1·03	2·48	1·41	1·25	1·85
1·009	0·811	8-3B	0·88	2·09	1·35	1·25	1·86

Table 4(c) $\xi = 15\%$

Period of shear building, T_{sb}	Period ratio, T_{sb}/T_{cm}	Frame type†	Response ratio for excitations				
			Q1	Q2	Q3	Q4	Q5
0·385	0·753	4-1C	1·90	0·81	0·83	0·94	0·72
0·395	0·708	4-2C	1·87	1·18	0·84	1·02	0·88
0·442	0·796	4-1B	1·69	1·07	0·78	0·93	0·81
0·453	0·740	4-2B	1·23	1·33	0·89	1·33	1·08
0·464	0·533	8-1D	2·00	1·33	0·89	1·72	1·69
0·466	0·809	4-3C	1·62	1·32	0·90	1·17	0·93
0·528	0·597	8-2D	1·84	1·48	1·10	2·05	2·23
0·534	0·841	4-3B	1·14	1·35	1·00	1·41	1·28
0·557	0·627	8-3D	1·92	1·46	1·23	1·96	2·43
0·595	0·869	4-1A	0·85	1·09	1·00	1·17	1·38
0·610	0·793	4-2A	1·13	1·16	1·11	1·27	1·39
0·721	0·900	4-3A	1·28	1·02	1·11	1·22	1·25
0·727	0·681	8-1C	2·43	1·87	1·43	1·51	1·59
0·832	0·744	8-2C	1·91	2·50	1·55	1·41	1·59
0·835	0·736	8-1B	1·70	2·37	1·46	1·32	1·50
0·880	0·772	8-3C	1·58	2·64	1·58	1·34	1·61
0·954	0·797	8-2B	1·09	2·38	1·55	0·37	1·69
1·009	0·811	8-3B	0·91	2·02	1·48	1·38	1·80

† The characters A, B, C and D represent column *EI* values of 19·2, 35·2, 45·8 and 118·8 MNm^2, respectively. Typically, 4-1C signifies a four-storey one-bay frame with column $EI = 45·8$ MNm^2.

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